



Listening to Children—Learning From Children

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ABSTRACT [\(Press Here for Sound\)](#)

This article reports the experience of a teacher and researcher who instituted a problem-posing component in her Grade 2/3 mathematics class with the help of a glove puppet, “Sylvester.” The activities, based largely on the intuition of an expert teacher, are substantiated by the literature on problem posing. The authors describe what happened in the classroom and some of the benefits of the strategy. Among the most important of these benefits was what the teacher learned about the children and their understanding of mathematics by listening to what they were saying.

Introduction

“It is no great secret that many people have a considerable fear of mathematics or at least a wish to establish a healthy distance from it.”

(Brown & Walter, 1990, p. 5)

All teachers of mathematics would like to reduce the distance between learners and the mathematics in their curriculum and to do so as early as possible in the children’s school careers. Children may develop skills in processing numbers mechanically but come to a halt when they are confronted with word problems. The knowledge that problem solving is improved when students pose their own problems is not new (Brown & Walter, 1990, 1993; Silverman, Winograd, & Strohauer, 1992), and there was considerable interest in the topic during the 1990s (English, 1997a, 1997b; Silver, 1997; Silver & Cai, 1996), supported by the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). Silver (1994) gave five benefits of having students pose their own problems: the connection

to creativity and exceptional mathematics ability; the improvement of students' problem solving; the ability to observe students' understanding of mathematics; to improve students' disposition towards mathematics; to develop autonomous learners. English (1997b) cites evidence that encouraging children to pose problems

can foster more diverse and flexible thinking, enhance students' problem-solving skills, broaden their perceptions of mathematics, and enrich and consolidate basic concepts....provide us with important insights into children's understanding of mathematical concepts and processes, as well as their perceptions of, and attitudes towards, problem solving and mathematics in general. (p.183)

But as Lowrie (2002) points out, most of the studies from the 1990s investigated children who were older than 10 years of age. Do younger children achieve the same benefits? Can young children compose mathematical problems? And how can children, especially young children, be given the opportunity and confidence to pose problems?

In this paper, we would like to share the experiences of Diane Sprackett, who set up her Grade 2/3 mathematics curriculum so that children could develop and solve their own problems in a non-threatening environment. We will describe what she did and by sharing Diane's words and showing some examples of the children's problems and their problem-solving strategies, we will describe some of the things we learned, and propose some benefits of her strategy. We will then propose that what Diane did from intuition has a well-established theory base, and that she foreshadowed the type of activity advocated in the recently revised curriculum in Quebec (Ministère de l'Éducation du Québec, 2001). This curriculum is based on a socio-constructivist approach and on the development of competencies within and across disciplines. Two of the cross-curricular competencies are problem solving and communication, and these are especially relevant in the mathematics curriculum from Kindergarten through Grade 11. Several years before the revised curriculum was implemented, Diane's classroom practice showed that she was encouraging children to construct their own knowledge, as well as breaking down the silos between disciplines.

Background

At the time Diane started the activities that we are describing, she had been teaching in elementary school for 18 years. She had always been especially interested in Language Arts, but two events prompted her interest in her students' learning of mathematics. A resource teacher was assigned to Diane's class to help her with some children with special needs and when the resource teacher probed individual children with questions like: "How did you get that answer?" Diane listened to some of the answers and recognized the variety of strategies that the children were bringing to the solution of mathematics problems. This made her conscious of the need to take the children's strategies into account to help her to assess their understanding. She had this in mind when she was asked to work on a committee to plan changes in the evaluation of mathematics in her school board's elementary schools. In her own class, she looked for ways to encourage children to talk about their mathematics and share the variety of their problem-solving strategies but she realized that sharing seemed to be much easier for them in story time than it was in mathematics.

Then came the Aha! experience. She had been using a teddy bear as a motivator for the children to create stories and thought, since the teddy bear had been so successful in creating stories, why not find an equivalent to help create mathematics questions? The idea was born—Sylvester the mathematician was bought!



Fig. 1: Sylvester

Early in the term, Diane modeled story problems that were relevant to the other class activities to give them ideas for contexts that might be fertile ground for mathematics problems. Then Diane said:

I'm tired of making up all these math problems, but Sylvester LOVES to make up problems. Would someone like to take home Sylvester the mathematician with his problem book tonight and make up a problem that we could solve tomorrow?

Children volunteered to take home Sylvester and an artist's pad so that Sylvester could make up a problem in his mathematics journal and the child could draw him and write out the problem and its solution. This became a pattern for the year and each child had the chance to set at least three problems during the year. There were two rules: The problem had to be one that the child knew how to solve, and he or she had to try it out on someone at home before bringing it to class. Unlike Brown (1984) and Silver (1994), Diane felt that posing the problem was not enough on its own: the child posing the problem had to be able to solve it as well. Sylvester quickly became so popular that Diane often had to help children plan their time by asking:

Is this really a good night for you? Or do you have Brownies? Swimming? Some other activity?

In mathematics class, the child (and Sylvester) read the question to the class, while Diane documented key information on the blackboard. The class asked extra questions for clarification and help, before trying to solve the problem. For example, Sara's¹ question was not clear enough:

My dad had 6 pens. I had 10. My mom had 7. And my sister stole 4 pens from all of us. How many are left?

Did her sister steal 4 pens? Or 12 pens? The children needed to ask appropriate questions and Sara needed to clarify her thoughts so that the rest of the class could understand and solve her problem.

Sometimes the problems became impossibly complex if "Sylvester" tried to show off. The confusion of coins and dollars in Figure 2 suggests that Jake was more interested in the story of "Jim and the Beanstalk" (Briggs, 1970) that the children had been reading in class than in posing a problem that anyone could answer:

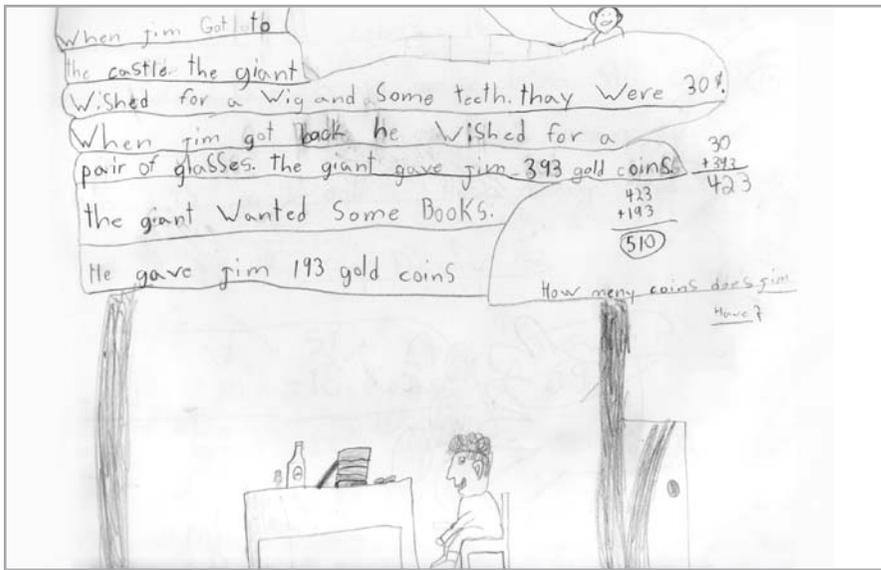


Fig. 2: Jake's first problem

When the problem was too complicated for the problem poser, Diane might say tactfully:

You and Sylvester sound like you're a little lost, do you want to have another go at this? Let's not make it so complicated that we stop having fun.

Then Sylvester went home for a second evening with no penalty to the child. The goal was to have children develop authentic problems that were mind-stretching, but not for the developer to show off. Even so, some of the problems demonstrated remarkably sophisticated mathematics.

After a second attempt, Jake's problem was much more amenable to solution (see Figure 3).

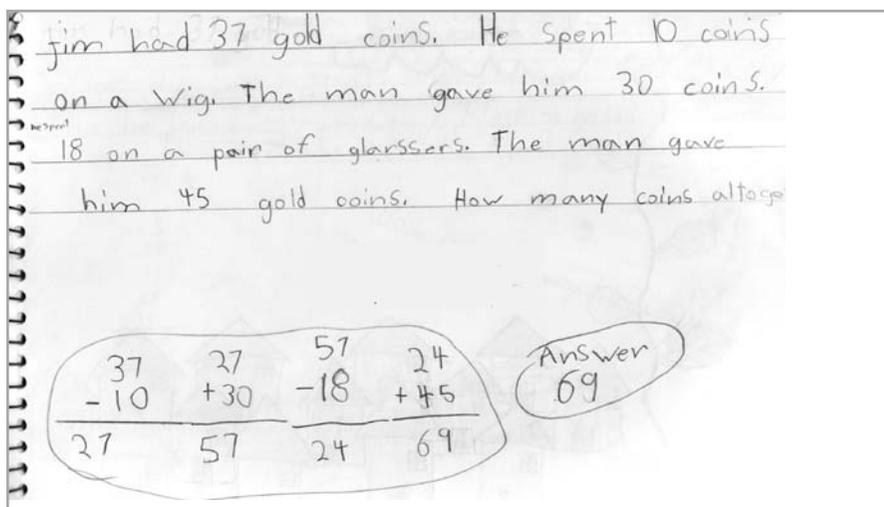


Fig. 3: Jake's second problem

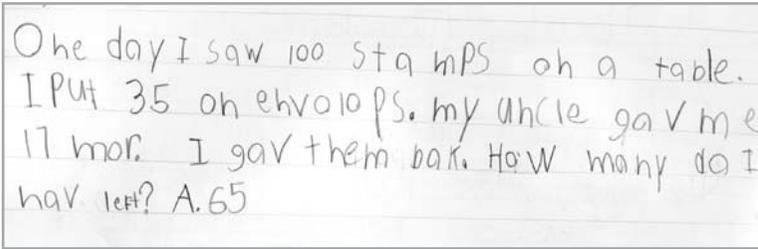
When the problem was defined clearly enough, the children usually returned to their tables where they worked in groups around a big sheet of newsprint to solve the problem. This encouraged children to work together and to talk to each other about what they were doing. Diane cut up these sheets of newsprint to add to each child's portfolios, and to give her information if she had not had the opportunity to talk to that particular child during the class. By using these sheets of newsprint in conjunction with the work they had done in Sylvester's mathematics journal, Diane was able to assess the children's progress during the year. Manipulative materials were always available, as well as a laminated hundreds chart and a number line at each table and on the classroom wall. Diane circulated and tried to give time to every child. At **the very** least she visited every table, to see that everybody was contributing, and to identify anyone who was completely lost. The following quotations are typical of her conversation with different children:

How could these manipulatives help you solve the problem? -- Find someone else at your table. Get them to explain to you how they did the problem. -- Show me another way you could do it.

After all the children had the opportunity to try the problem, they would reconvene to discuss their solutions and share their strategies with the whole class. Then Diane's conversation included questions like:

Who tried it a different way? -- We'll talk about the answer eventually, but what I'm most interested in is HOW you got your answer. -- Or if you haven't got the final answer yet, I'm interested in what you've done so far and how far you've got.

Some children became good at including red herrings in their problems, but the class also became good at detecting them. Brian's extra 17 stamps did not confuse them for long (see Figure 4),



One day I saw 100 stamps on a table.
I put 35 on envelopes. my uncle gave me
17 more. I gave them back. How many do I
hav left? A. 65

Fig. 4: Brian's stamp problem

Clearly, they did not confuse Brian either:

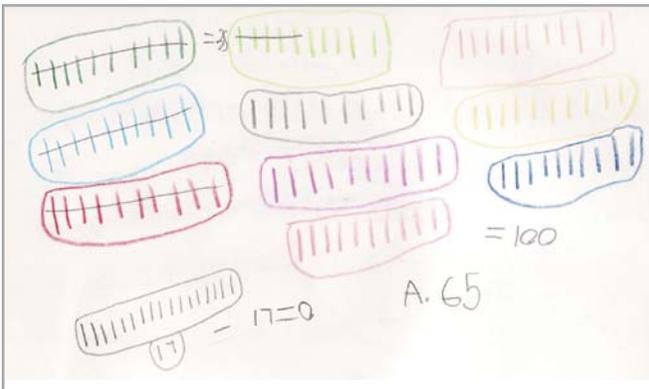


Fig. 5: Brian's solution

In Brian's solution, we can see that he is well on the way to making the transitions among using manipulatives, translating the manipulatives into symbols, and translating the symbols into numbers. Diane's relaxed and extensive use of manipulatives (LEGO®, straws, plastic bag ties, bottle caps, etcetera) made this transition a natural process that individual children could undertake when they were ready, not according to an arbitrary date.

Being very sensitive to the fragile egos of young mathematicians, Diane used much indirect teaching and many gentle offers of help:

Would you like me to help you find another way to solve this?

But she made a mistake when she criticized Daniel's problem.

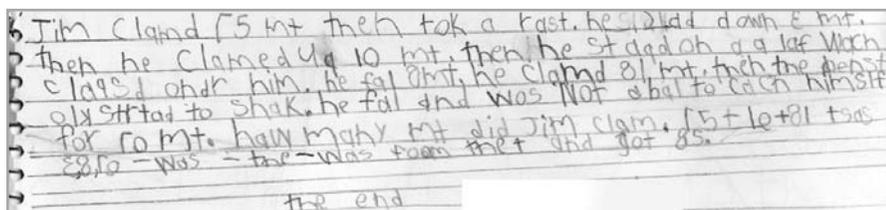


Fig. 6: Daniel's difficult problem

Daniel was new to classroom life, having been home-schooled for a year. He began the year as a non-reader and his invented spelling made his work very difficult to understand, so we will “translate” his first problem that was based on “Jim and the Beanstalk.”

Jim climbed 75 m then took a rest. He slid down 3 m then he climbed up 10 m. Then he stood on a leaf which collapsed under him. He fell 8 m then the beanstalk started to shake. He fell and was not able to catch himself for 70 m. How many metres did Jim climb?

His answer could be translated as:

75 + 10 + 81 pluses. 3, 8, 70 minuses. Minus the minuses from the plus and get 85,

but in the bustle of the class, Diane could not do the necessary translation quickly enough, and suggested that Daniel try again. An angry and upset Daniel went home saying: “SHE didn’t understand it.” In his next problem, his father helped him to lay out the answer so that Diane *could* understand it.

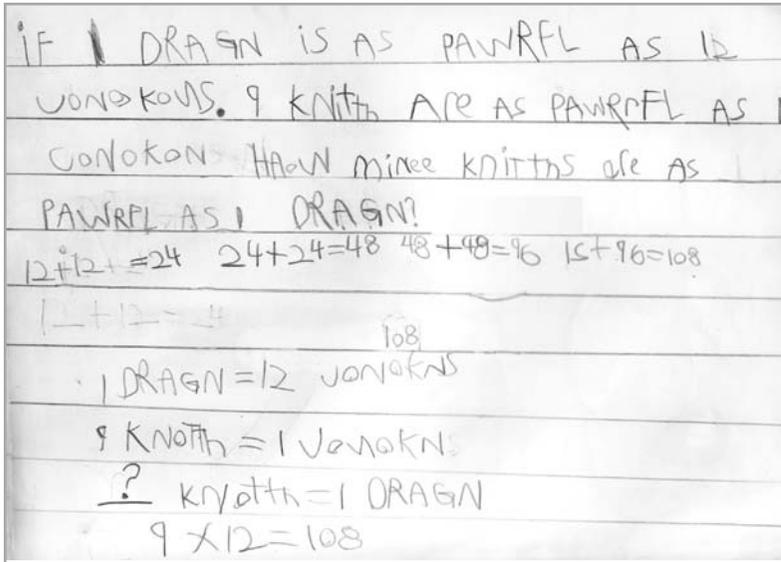


Fig. 7: Daniel's second problem

Again we will attempt a translation of Daniel's words:

If 1 dragon is as powerful as 12 vongkons (?) and 9 kittens are as powerful as 1 vongkon, how many kittens are as powerful as 1 dragon?

$$12 + 12 = 24 \quad 24 + 24 = 48 \quad 48 + 48 = 96 \quad 12 + 96 = 108$$

$$1 \text{ dragon} = 12 \text{ vongkons}$$

$$9 \text{ kittens} = 1 \text{ vongkon}$$

$$? \text{ kittens} = 1 \text{ dragon}$$

$$9 \times 12 = 108$$

Daniel's solution shows that although he could not yet multiply 9×12 (the last lines were added with his father's help), he had an amazingly sophisticated grasp of algebra and ratio and could keep track of successive additions. The message from Daniel's work was: ask children how they got an answer and then listen carefully to their explanations to find out what mathematics they know.

Because this was a multi-age, multi-level class, Diane was dealing with a wide range of abilities and problem-posing skills, from Sara to Daniel and Brian. Like Ellerton (1986), Diane found that the more able children developed, talked about, and solved more complex problems than weaker students, and could communicate

about them better in conversation. The range of problem difficulty provided insights into the abilities of the problem-positors as well as the solvers and Diane decided that occasionally giving children problems that are beyond their abilities gives a lot of information regarding their *understanding* of mathematics. Unlike many of the mechanical exercises found in textbooks, they could not solve Sylvester's problems by simply taking a recently learned algorithm and applying it to two numbers. Yet they could often develop their own algorithms and apply them appropriately.

Benefits of This Approach

"I do believe that problems are the heart of mathematics, and I hope that as teachers... we will train our students to be better problem posers and problem solvers than we are." (Halmos, 1980, p. 524)

In allowing so much time for developing good problem posers and solvers, other aspects of mathematics were not neglected. There was still a place in the daily mathematics lessons for learning strategies, dealing with misconceptions and using the textbook as a resource, but as Diane reflected on the children and their problem posing, and as we later discussed her strategies, many benefits became clear to us, making the time spent worthwhile. At a superficial level, children were learning to be realistic in planning the time available to them when they volunteered to take Sylvester home. On a deeper level, they were taking the initiative in their own learning and setting the agenda for their mathematics class. We will list some of the more salient advantages that we observed with more examples of the students' work.

1. Connections with the child's world

Modeling questions at the beginning of the year had led Diane to ask—and to propose an answer to—the question:

What's a real problem? -- The standard textbook example is: "If you went to the store..." or "...Farmer Brown has 15 cows..."

It's better, because it's more relevant to the children, to start with: "The lunchroom can only take x children. Can it hold Mrs Gray's class and Mr Hackett's class and ...?" or "Camp is going to cost \$40 each...." You know they're doing math, they think they're just going to camp.

In brief, we should be conscious of things that are happening around the children that are mathematics problems and that are relevant to them, rather than relying on problems contrived by textbook authors for generic classrooms. Having them compose their own problems was a way to allow children to draw from their own worlds. When the child has made the connection with his or her own world, the problems are truly an intellectual challenge that is worth accepting, or an “intrinsic motivation” (Kilpatrick, 1985).

2. Children develop their own strategies according to their own abilities

There are many instances from Diane’s class of children developing their own methods for solving their problems, showing that they are truly constructing their own knowledge. We will give two examples:

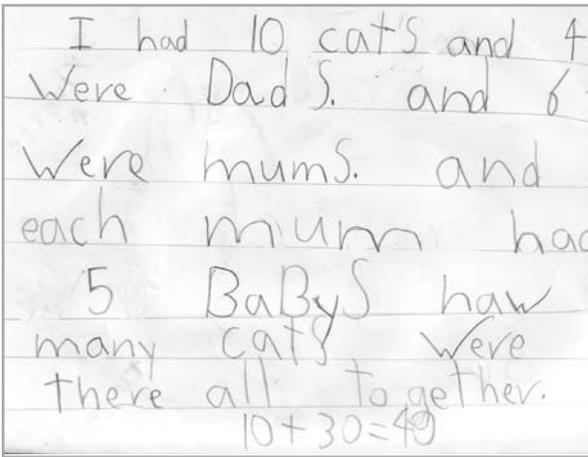


Fig. 8: Jake’s baby cats

On the page that gave the solution to this problem, Jake had solved the problem of multiplying 5 by 6—an operation he had not yet learned formally—by drawing six cats’ heads and making five dashes alongside each of them.

Paolo’s solution to his problem is an impressive indicator of his sense of place value and his ability to manipulate 3-digit subtraction, again, an operation that had not been taught to the class.

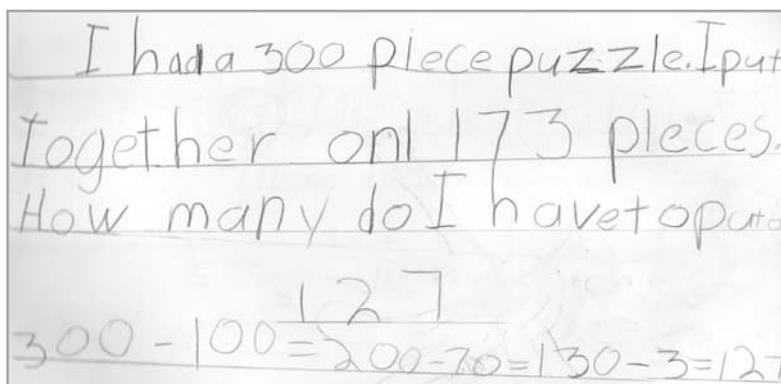


Fig. 9: Paolo's solution

3. Process, not product is emphasized

Because Diane's class had a wide range of ages and abilities, some children set 4-step problems that others could not even approach. When they first listened to the problem, Diane observed the children's reactions and asked

Who's feeling totally overwhelmed?

Children were quite honest about where they were in mathematics, so they made the decision about how much they could do. Then weaker students were told:

Just go this far.

Children were not upset by this, as Diane constantly emphasized:

I'm looking for your strategies. You don't have to get an answer.

But her method also gave the opportunity for some students to shine. As well as children like Sara and Jake, whose work has been shown earlier, there were also children like Bora, whose first problem has tricked many adults who have seen it:

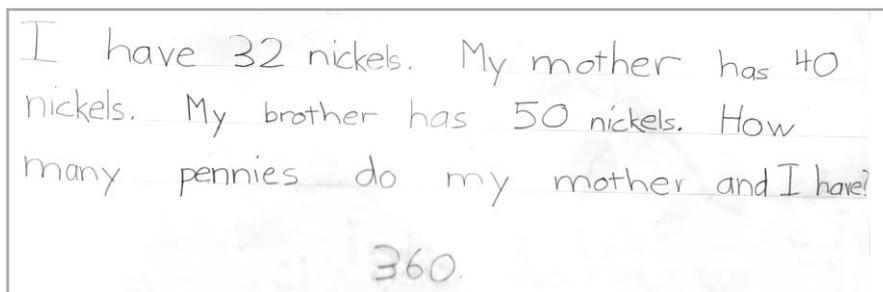


Fig. 10: Bora's first problem

It would be difficult to find a textbook with a range of problems that would serve such a range of children, yet teachers are faced with this spectrum of abilities on a daily basis.

4. Assessment is authentic

Walking around and observing the children's work at their tables gave Diane an overview of their abilities, and she recorded these observations on sticky notes to add to their portfolios and supplement their newsprint solutions. Most interesting was the opportunity to note how different children saw the same problem and applied different strategies. By emphasizing process, and by encouraging different solution strategies, Diane was able to probe the children's understanding and to assess their knowledge of mathematics.

5. Learning in a group

In the regular classroom setting, the teacher is interacting with one child, and the others are not necessarily engaged. When they are grouped around a child solving his or her problem, or working around a table on one large piece of paper, there is much more involvement and learning is socially constructed.

6. Communication among students

It is clear that children are communicating in a variety of ways in these classes. They are listening to the question being read by its creator (and Sylvester); they are asking for clarification and the problem poser has to clarify; they are sometimes working together on solutions; and they are listening to each other's explanations and descriptions of strategies.

7. Communication with parents

The children were required to try out the problem with somebody at home to resolve any initial difficulties. Pierre had behaviour problems as well as difficulty in processing numbers and he struggled in mathematics, causing him to sit for half a year with his arms folded saying, "I don't know." Often, he could not see the first step in a problem because there was too much information, too many numbers. Sometimes Diane substituted smaller numbers, sometimes modeled for him with manipulatives. Pierre benefitted from much support at home. His father said that he would help him to set easy problems, but Pierre wanted to set more difficult ones. He

was aware of the level the other children were working at and knew it would improve the other children's poor image of him if he could do the same. So his father helped him organize and structure his problems. After a while, Pierre presented this problem:

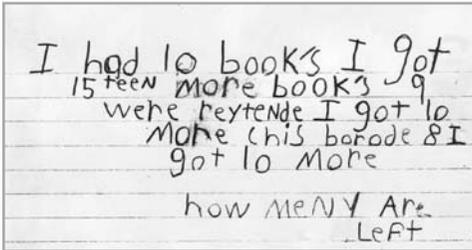


Fig. 11: Pierre's problem

I had 10 books. I got 15 more books. 9 were returned. I got 10 more. Chris borrowed 8. I got 10 more. How many are left?

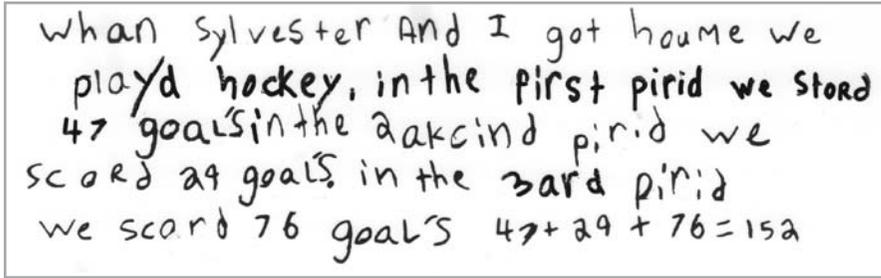


Fig. 12: Pierre's solution

Gradually, with his father's help at home, Pierre's self-confidence improved by developing a problem that challenged his classmates, and by reading his problem to the class. This improvement in his self-esteem could not have happened without communication between Diane and Pierre's family, and this communication helped the adults to understand what Pierre was feeling and gave him the support that helped him to succeed.

8. Helps children learn what a problem is

We sometimes forget that many young children are not clear what a problem is. For children like Dina, the problem is finished when they have created the situation and given the facts, and they often forget to pose the question.



When Sylvester and I got home we
 played hockey, in the first period we scored
 47 goals in the 2nd period we
 scored 29 goals in the 3rd period
 we scored 76 goals $47 + 29 + 76 = 152$

Fig. 13: Dina's hockey "problem"

These are the children who often respond to assigned problems with, "I don't understand the question!" Giving them the opportunity to create their own questions and present the problem to their peers is a valuable learning tool.

9. Mathematics and other disciplines

Sylvester also showed the child's ability with writing in areas such as spelling, capitalization, and punctuation. Children were not penalized for weakness in language, yet it transpired that children were not generally weak in just one area, so the problems they developed were often representative of their other abilities, and gave Diane insight into their other abilities.

Many children's books have an unexpected amount of mathematics content. When the class had read "Jim and the Beanstalk" (Briggs, 1970), this gave another springboard for children's mathematics problems when Jim did such things as measure the giant's mouth for new false teeth and his head for a wig (see, for example, Jake's problems in Figures 2 and 3 and Daniel's in Figure 6). Like Lowrie (2002), we found that most of the children's problems were based on knowledge of number concepts, even when they could have involved other mathematical strands, perhaps because of the emphasis on number in the early grades curriculum.

10. Children's enjoyment

Diane described Anna as a waif. She came from a troubled home and had no self-confidence. She was the last child to take Sylvester home, and her first problem was almost a copy of the one before it in Sylvester's journal. Later in the year, she was able to set a problem like the one shown in Figure 14 and even proposed a realistic solution for dealing with the remainder. Halfway through the year, Diane was overjoyed to hear Anna say: "Math is fun!"

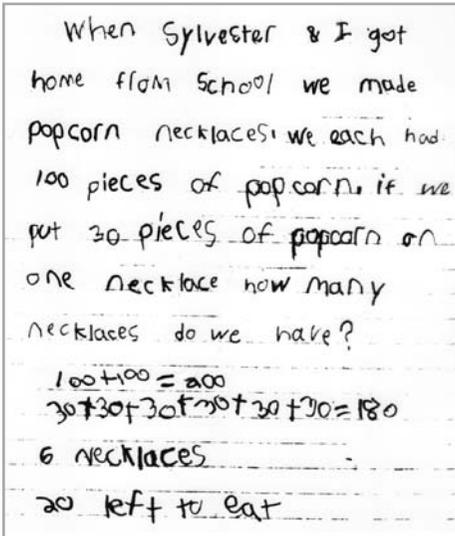


Fig. 14: Anna's popcorn problem

Part of the children's enjoyment came from the way that any anxiety about mathematics was reduced by using the glove puppet as an alter ego and mouth-piece. If the problem was too hard, or the child's solution was wrong, Sylvester could be the culprit, while the child could take credit for good work. This benefit of student enjoyment cannot be overestimated. Ownership of the problem to be solved—and indirectly, ownership of the curriculum for that lesson—leads to enjoyment, which leads to success.

11. Opportunity for the children to design the curriculum

Eventually, Sylvester became too popular, the discussion of the variety of strategies was taking a long time; the children said that it would be better if Diane did not listen to so many solution strategies. By the time it came to share solutions, the

children had already discussed strategies with their groups, and Diane had already talked and listened to a number of students. This led her to perform a judicious sampling of strategies that gave voice to a wide range of abilities, but that did not take too long. In this way, the quicker students were exposed to alternative strategies and the slower students could listen to a range of strategies that often allowed them to develop a strategy of their own. It also gave the children the opportunity to have input into their curriculum and for Diane to modify her plans to accommodate their opinions.

12. Listening to children

Finally, we emphasize an issue that has pervaded this whole description—Diane’s use of problem posing, helped by the use of a surrogate in the form of Sylvester, gave the children the freedom to speak and her the opportunity to listen to what the children were saying. This gave Diane insights into their thinking and learning so that she could modify her teaching to meet their needs.

Summary

The program is organized around three competencies: the first refers to the ability to solve situational problems; the second pertains to mathematical reasoning, which implies familiarity with concepts and processes specific to mathematics; and the third focuses on communication using mathematical language. (Ministère de l’Éducation du Québec, 2001, p. 140)

This extract from the mathematics curriculum for Quebec elementary schools might have been written to describe the activities Diane had previously developed for her class. The children’s problems were developed within “situations” that were relevant to them. Mathematical reasoning, sometimes previously learned, sometimes developed by the child, is clearly apparent in the children’s work. All aspects of the classroom communication were based on correct mathematical language.

The program of studies in Quebec has been revised to incorporate socio-constructivist methodology and cross-curricular competencies, one of which is problem solving. In this, it is similar to initiatives in other jurisdictions, putting emphasis on student-initiated learning. Problem solving has been a focus of worldwide

mathematics education since the 1980s and continues to interest teachers and researchers. In the upcoming conference of the International Commission for the Study of Improvement of Mathematics Education (ICSIME) (slated for July 2009), two of the themes to be investigated are “Problem solving and institutionalization of knowledge,” based on Lakatos’ (1976) statement that mathematics is a dialogue between individuals who have problems to solve, and “Creativity in mathematical activities,” supporting the assertion that:

Mathematical creativity and innovation are often cited as critical to success in work and in life in this twenty-first century world. Teachers, mathematics educators, mathematicians, researchers, parents, and students themselves all have a stake in learning how best to nurture and support this development of mathematical creativity and the realization of mathematical promise. (ICSIME, 2009)

Diane’s initiative in using Sylvester to support her children’s creativity in problem solving surely epitomizes these themes and her activities in mathematics classes would be as current today as they were innovative when she was doing them. We will end with some more of Diane’s reflections about what she did:

Sometimes we get so tied up with following the curriculum, and we get nervous about what we’re leaving out, but when you listen to what the kids are showing us they can do, they are setting the curriculum and we don’t realize that kids can do that. We teachers think we’re the ones who have to do it all.

I feel I’m addressing the children’s needs more at this time than at any other time in math class. I feel that I learn more about the children’s mathematical ability by watching them solve problems than by any other method. Open-ended questions allow that kind of scope, then I hate going back to the textbook questions....I’m learning from the kids about how kids learn math....If we keep listening to the kids, they tell us how to teach them.

And finally, during one session where strategies were shared, Diane told her class what any teacher would love to be able to say at the end of a lesson:

Wow! Now I’ve got thinkers!

Notes

1. This, and all the children's names, are pseudonyms.

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Diane Sprckett was a teacher with the Lester B. Pearson School Board in Quebec for 21 years. She has taught all elementary grades from pre-K to grade 6. While Language Arts was always her passion, she discovered the joy of observing the mathematical thinking of young children.